Optimising Container Stowage: Minimising Relocations in Maritime Logistics

BSc. in Applied Mathematics - IE University

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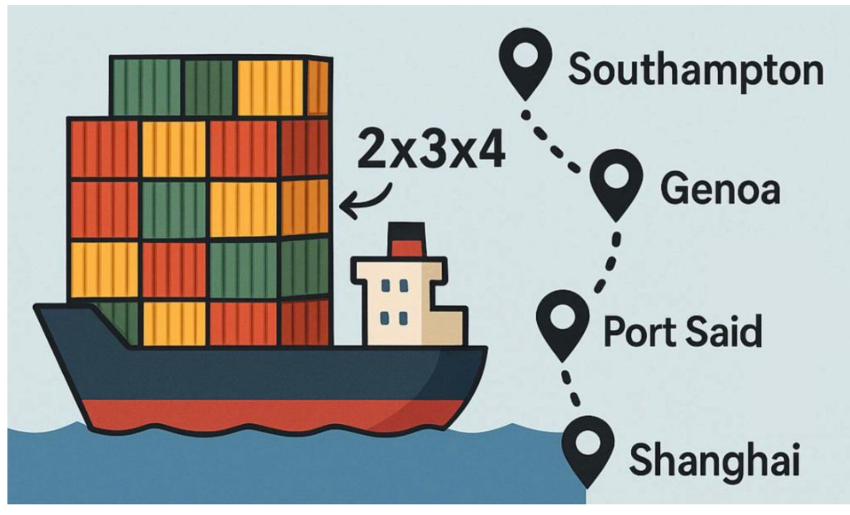
Introduction

Container shipping plays a crucial role in global trade, enabling efficient movement of goods between ports worldwide. In container shipping, vessels visit several ports to load and unload containers. The arrangement of containers within the ship affects how efficiently these operations can be performed. Poor planning can lead to relocations, where containers must be temporarily moved to reach others beneath them. These extra movements increase handling time and costs. We aim to develop an effective stowage plan that minimizes relocations and optimizes the efficiency of port operations throughout the voyage (Taha, 2017).



Methodology

Sets:

$$\mathcal{I} := \text{set of containers (index } i\text{)},$$
$$\mathcal{P} := \text{set of ports (index } p\text{)},$$
$$\mathcal{S} := \text{set of stacks (index } s\text{)},$$
$$\mathcal{T} := \text{set of tiers (index } t\text{)}.$$

Data:

$$M_i := \text{maximum number of containers allowed above container } i \quad (i \in \mathcal{I}).$$

Decision variables:

$$r_{ip} \in \begin{cases} 1, & \text{if container } i \text{ is relocated at port } p, \\ 0, & \text{otherwise;} \end{cases}$$

$$x_{istp} \in \begin{cases} 1, & \text{if container } i \text{ is stored in stack } s \text{ at tier } t \text{ after port } p, \\ 0, & \text{otherwise;} \end{cases}$$

$$\ell_{ip} \in \begin{cases} 1, & \text{if container } i \text{ is loaded at port } p, \\ 0, & \text{otherwise;} \end{cases}$$

$$u_{ip} \in \begin{cases} 1, & \text{if container } i \text{ is unloaded at port } p, \\ 0, & \text{otherwise;} \end{cases}$$

$$y_{ip} \in \begin{cases} 1, & \text{if container } i \text{ is on board after completing operations at port } p, \\ 0, & \text{otherwise;} \end{cases}$$

Objective: Minimise total relocations

$$\min \sum_{i \in \mathcal{I}} \sum_{p \in \mathcal{P}} r_{ip}$$

Constraints:

(1) Tier below filled

$$\sum_{i \in \mathcal{I}} x_{istp} \geq \sum_{i \in \mathcal{I}} x_{is(t+1)p}, \qquad \forall\, s,\ t,\ p$$

(2) Max containers above when i is at (s,t)

$$\sum_{r=t+1}^{4} \sum_{j \in \mathcal{I}} x_{jsrp} \leq M_i + 4(1 - x_{istp}), \qquad \forall\, i, s, t, p$$

(3) On-board indicator consistency

$$\sum_{s \in \mathcal{S}, t \in \mathcal{T}} x_{istp} = y_{ip}, \qquad \forall\, i, p$$

(4) Slot capacity

$$\sum_{i \in \mathcal{I}} x_{istp} \leq 1, \qquad \forall\, s, t, p$$

(5) Relocation consistency (load relation)

$$r_{ip} \geq x_{istp} - x_{ist(p-1)} - \ell_{ip}, \qquad \forall\, i, s, t, p = 2, \dots, |\mathcal{P}|$$

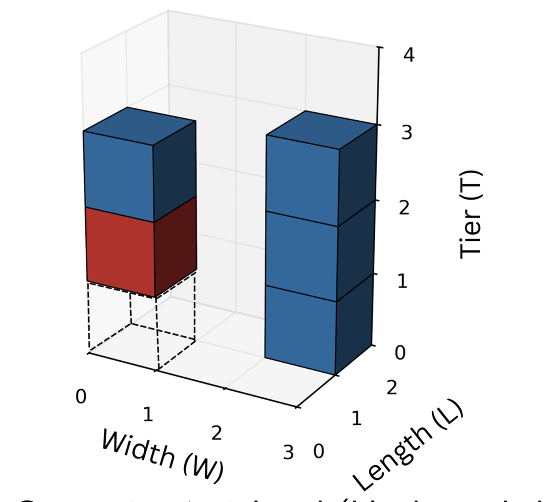
(6) Relocation consistency (unload relation)

$$r_{ip} \geq x_{ist(p-1)} - x_{istp} - u_{ip}, \qquad \forall\, i, s, t, p = 2, \dots, |\mathcal{P}|$$

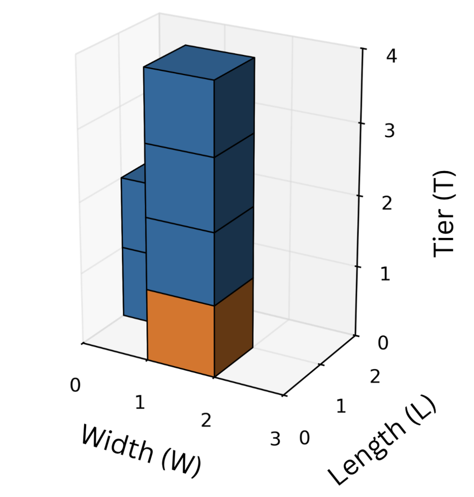
(7) Variable domains.

$$r_{ip},\ x_{istp},\ \ell_{ip},\ u_{ip},\ y_{ip} \in \{0, 1\} \qquad \forall i \in \mathcal{I},\ s \in \mathcal{S},\ t \in \mathcal{T},\ p \in \mathcal{P}.$$

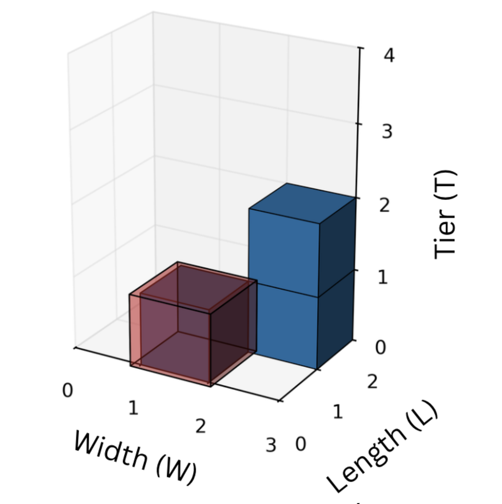
Constraint Visualisations



Constraint 1 violated (block needed below the red block)



Constraint 2 violated (max above orange block = 2)

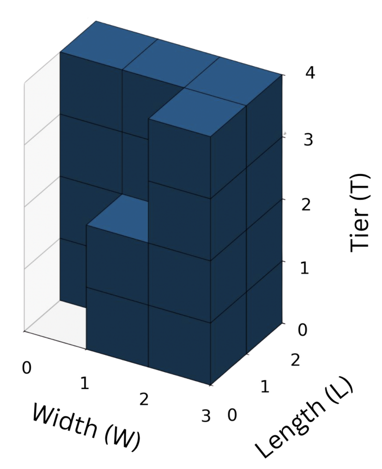


Constraint 4 violated (only one block allowed in each slot)

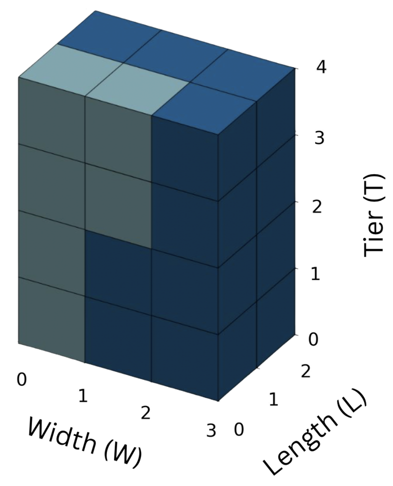
Results

Our optimisation model produced a fully optimal stowage plan, requiring only two relocations across the entire journey, both at Port 3 (see figures below). The final solution was computed in 44.69 seconds using the HiGHS (Phan et al., 2021) MILP solver on a standard MacBook Pro (M4 Pro, 10-core GPU, 16-core CPU), showing that high-quality planning can be achieved quickly without specialised computing equipment. The only required inputs were ship dimensions and basic container information (origin, destination, and stacking limits), making the method easily generalisable to different vessel constraints or routes.

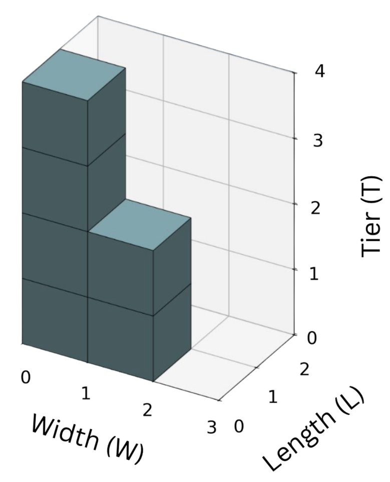
From an operational perspective, the results offer clear and actionable insight: only Port 3 requires a small, planned reshuffle, while all remaining ports can load and unload without interruption. This leads to greater time predictability, better equipment and labor planning, and reduced risk of unexpected delays during port calls. During development, we also ensured the software remained easy to scale by keeping all data flexible rather than hard-coded. If new requirements are needed, users would only need to update the constraints or objective function rather than rebuild the system.



Port 1



Port 2



Port 3

Further Improvements

The current formulation is unlikely to scale efficiently to much larger problems without additional computing power or a more advanced solver to maintain reasonable run times. As shown by Parreño-Torres, Alvarez-Valdés, and Parreño (2019) (Parreño-Torres et al., 2019) , even moderate increases in problem size require powerful exact solvers such as CPLEX or the use of heuristics like GRASP to obtain solutions within practical time limits.

Conclusions

Our optimisation model achieved an optimal solution (Bradley et al., 1977), confirming that the method can effectively minimise unnecessary container relocations during port operations. By planning how containers are placed on the vessel more carefully, it is possible to reduce extra moving costs, save time, reduce handling costs at ports, and be more sustainable. In the future, this method could be applied to larger shipping routes and more complex systems to further improve efficiency and support smoother port operations.

References

Bradley, Stephen P., Arnoldo C. Hax, and Thomas L. Magnanti. *Applied Mathematical Programming*. McGraw-Hill, 1977.

Parreño-Torres, Consuelo, Ramon Alvarez-Valdés, and Francisco Parreño. "Solution Strategies for a Multiport Container Ship Stowage Problem." *Mathematical Problems in Engineering* 2019 (2019): Article ID 9029267, 12 pages.

Phan, D., J. Hall, and M. Trick. 2021 "HiGHS: A High Performance Serial and Parallel Linear Programming Solver.". Optimization Online.

Taha, Hamdy A. *Operations Research: An Introduction*. 10 edition. Pearson, 2017.