Multicommodity Network Design

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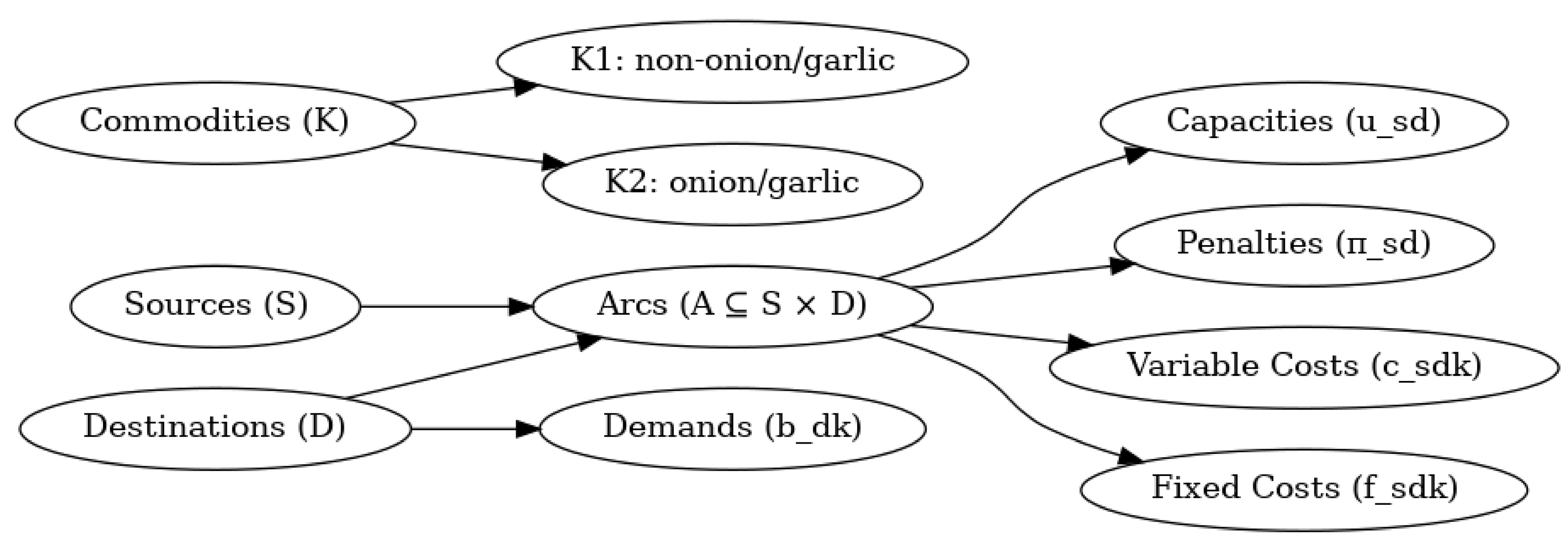
Introduction

Goal. Determine a weekly shipment plan that satisfies all demands at minimum cost while: obeying arc capacities; paying fixed and variable shipping costs; enforcing food safety: odorous food cannot travel on the same arc with other produce; ensuring reliability: each (destination, commodity) is sourced from ≥ 2 origins, and charging a one-time compatibility penalty on an arc if ≥ 2 commodities use it.

Context. Nodes:



Methodology

Decision Variables

- $x_{sdk} \geq 0$: flow of commodity $k \in \mathcal{K}$ on arc $(s,d) \in \mathcal{A}$.
- $y_{sdk} \in \{0,1\}$: 1 if commodity k uses arc (s,d) (triggers f_{sdk}).
- $z_{sd} \in \{0,1\}$: 1 if ≥ 2 different commodities use arc (s,d) (triggers π_{sd}).
- $g_{sd}^{(1)}, g_{sd}^{(2)} \in \{0,1\}$: binary indicators of group 1 or group 2 on arc (s,d) (for incompatibility).

Model

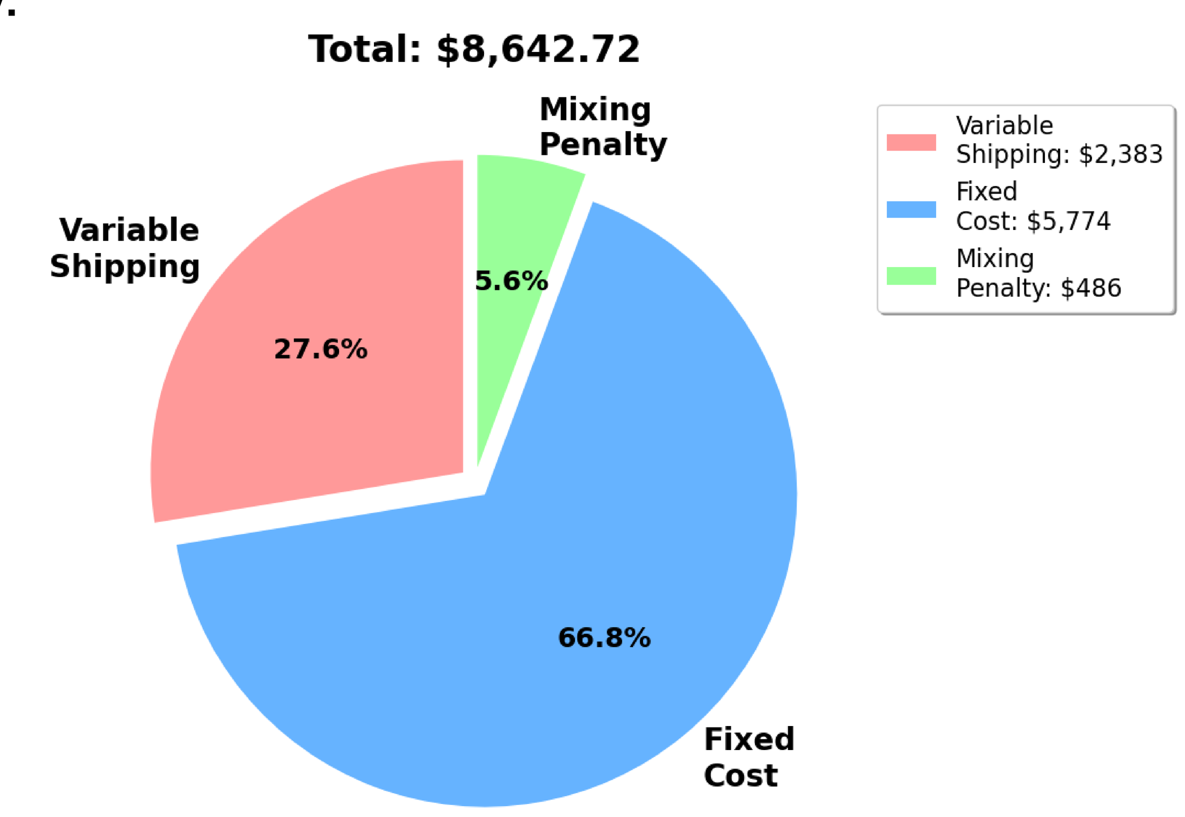
$$
\begin{aligned}
\min \quad & \sum_{(s,d)\in\mathcal{A}} \sum_{k\in\mathcal{K}} (c_{sdk}x_{sdk} + f_{sdk}y_{sdk}) + \sum_{(s,d)\in\mathcal{A}} \pi_{sd}z_{sd} \\
\text{s.t.} \quad & \sum_{s:(s,d)\in\mathcal{A}} x_{sdk} = b_{dk}, \forall d \in \mathcal{D},\ k \in \mathcal{K} \\
& \sum_{k\in\mathcal{K}} x_{sdk} \leq u_{sd}, \forall (s,d) \in \mathcal{A} \\
& x_{sdk} \leq u_{sd}y_{sdk}, \forall (s,d) \in \mathcal{A},\ k \in \mathcal{K} \\
& \sum_{s:(s,d)\in\mathcal{A}} y_{sdk} \geq 2, \forall d \in \mathcal{D},\ k \in \mathcal{K} \\
& g_{sd}^{(1)} \geq y_{sdk}, \forall k \in \mathcal{K}_1,\ (s,d) \in \mathcal{A} \\
& g_{sd}^{(2)} \geq y_{sdk}, \forall k \in \mathcal{K}_2,\ (s,d) \in \mathcal{A} \\
& g_{sd}^{(1)} + g_{sd}^{(2)} \leq 1, \forall (s,d) \in \mathcal{A} \\
& z_{sd} \geq y_{sdk_1} + y_{sdk_2} - 1, \forall (s,d) \in \mathcal{A},\ k_1 < k_2 \\
& x_{sdk} \geq 0,\ y_{sdk} \in \{0,1\},\ z_{sd} \in \{0,1\}, \\
& g_{sd}^{(1)},\ g_{sd}^{(2)} \in \{0,1\}.
\end{aligned}
$$

Results

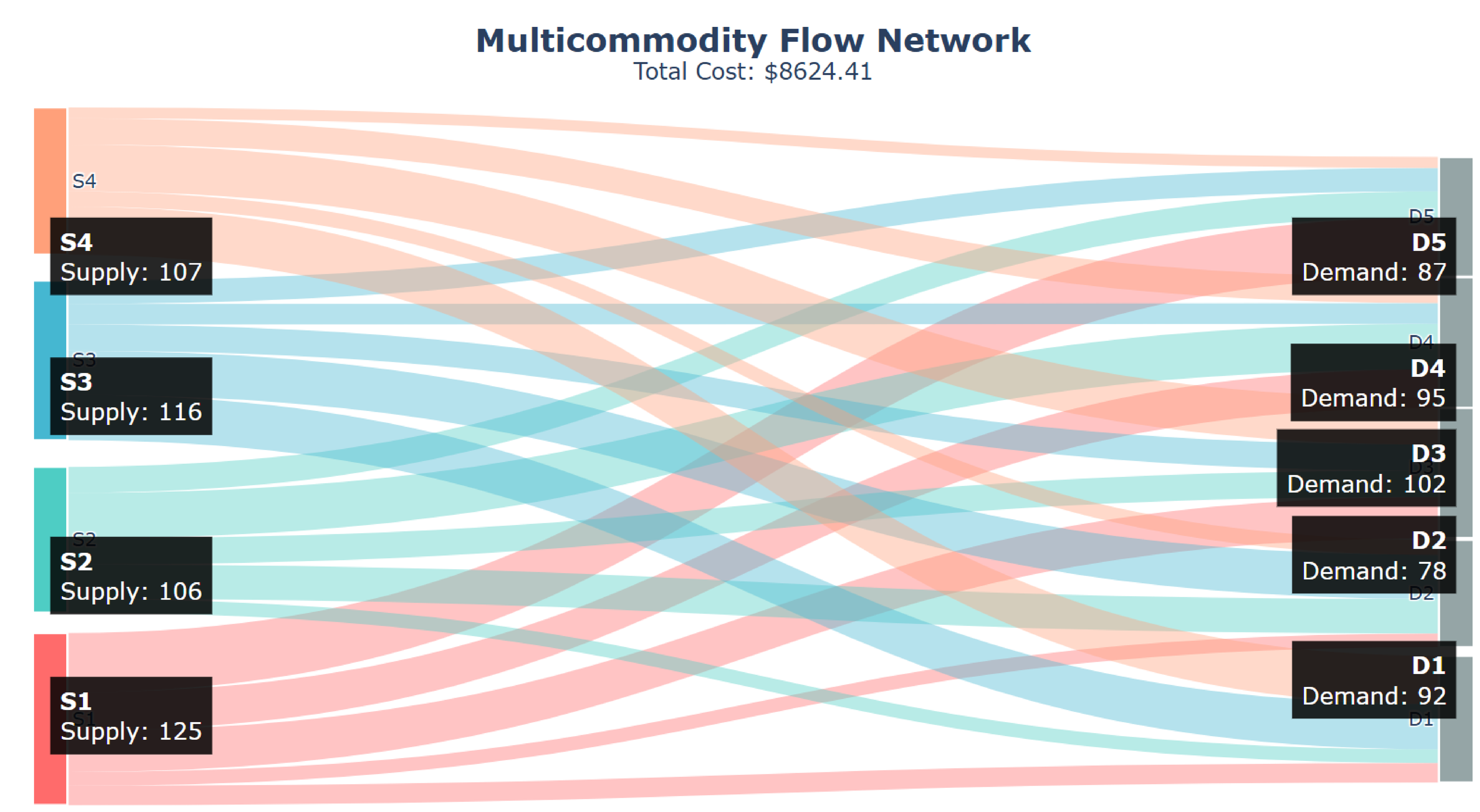
Key Metrics

Solver: Solved with HiGHS[1] (time limit = 60 s, MIP gap = 1%).

Objective value: 8 642.7, obtained on a system with 8 GB RAM and an Intel Core i5 processor. The solver reached an optimal solution within the set limit, offering a good balance between computational efficiency and solution quality.



Total cost $8 642.7: fixed cost 66.8%, variable 27.8%, penalty 5.6%.



Illustrates the complete flow of commodities from sources (left) to destinations (right), with arc thickness proportional to shipment volume.

Conclusion

One-time penalties shape arc mixing: higher penalties limit commodity mixing and can raise fixed costs. Diversification increases cost and robustness. Future work can include adding seasonality, peak-demand effects, and correlations between produce types.

References

[1] Huangfu, Qi, and Julian Hall. "Parallelizing the Dual Revised Simplex Method." *Mathematical Programming Computation*, vol. 10, no. 1, 2018, pp. 119–142. SpringerLink, https://doi.org/10.1007/s12532-