LINEAR ALGEBRA

Grado en Matemáticas Aplicadas / Bachelor in Applied Mathematics BAM SEP-2025 LA-AM.1.M.A

Area Mathematics

Number of sessions: 35

Academic year: 25-26

Degree course: FIRST

Number of credits: 6.0

Semester: 1º

Category: BASIC

Language: English

Professor: **IRENE ALDA FERRERO**

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Irene Alda started to do research as a young physicist before and during her undergraduate studies. After completing her MSc. in Advanced Materials, she started a Ph.D. in Physics—technically in Photonics—at the Institute of Photonic Sciences (ICFO) with her own funding (FPU scholarship). Irene's research project was part of the ERC consolidator grant QnanoMECA in the group of ICREA Professor Dr. Romain Quidant. She graduated in 2020 "Cum Laude" with her dissertation: "Levitodynamics on-a-chip: from planar Paul traps to near-field optical nanocavities". During this time, she co-led teaching activities for undergraduate students from the Universitat Politècnica de Catalunya. In 2023, she joined IE University as the Academic Director of the Bachelor in Applied Mathematics. She ensures academic quality, excellent student experience, and effective management of the undergraduate program, plus, she is also involved in teaching activities.

Education:

- Ph.D in Photonics, ICFO & Universitat Politècnica de Catalunya.
- MSc. in Advanced Materials, Universidad Autónoma de Madrid.
- BSc. in Physics, Universidad Autónoma de Madrid.

Office Hours

Office hours will be on request. Please contact at:

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SUBJECT DESCRIPTION

This course covers the essentials of linear algebra, including applications in physics, natural sciences, and engineering. You'll start with the basics of vector spaces and matrix operations and learn to solve systems of linear equations using Gaussian elimination. As the course progresses, topics like linear transformations, determinants, and their properties are explored. The curriculum progresses from fundamental techniques to advanced concepts such as diagonalization, the Jordan canonical form, and singular value decomposition.

Each module incorporates real-world examples to solidify your understanding and application of linear algebra in solving complex challenges. By the end of this course, you will have a thorough grasp of linear algebra and be well-prepared to apply these concepts effectively across various scientific and engineering disciplines.

LEARNING OBJECTIVES

- Demonstrate knowledge and understanding of concepts related to vector spaces and vector subspaces (vector independence, bases, dimensions, intersection, union, and sum).
- Understand the concepts related to linear transformations (e.g., kernel, image).
- Perform matrix computations and use matrix operations to efficiently solve linear systems (Gauss elimination) and determine the inverse-
- Apply the properties of determinants to solve problems and use them appropriately in proofs and problems.
- Demonstrate proficiency in finding eigenvalues and eigenvectors of a matrix and determining if a matrix is diagonalizable.
- Demonstrate knowledge and understanding of the Jordan form and finding generalized eigenvectors.
- Demonstrate knowledge about the Cayley-Hamilton Theorem, characteristic polynomial, and minimal polynomial.
- Understand Singular Value Decomposition and applications in linear algebra.

TEACHING METHODOLOGY

Before each class, students are expected to work on assignments and readings at home. The course lectures will cover both theoretical explanations and practical exercises. Each mathematical concept will be introduced or followed by one or more examples. Students need to participate in class to acquire the skills needed to solve exercises and homework problems. Webwork problems, advanced problem sets, and quizzes will be used throughout the course to assess student progress.

Webwork problems and advanced problem sets will help students understand and deepen their knowledge of the material covered in class. It is strongly recommended that students complete the homework exercises and the additional proposed problems during the course and not leave them for a date close to the exam. While students are encouraged to work with others on understanding the lecture material and attempting assignments, it is important that students can complete these problems on their own to prepare for the final exam.

Quizzes will be given throughout the semester, covering previously taught material. These quizzes are meant to test the overall understanding of the material and help the professor assess the overall performance and evolution of the class. The course will also include one **midterm exam** and a three-hour **final exam**.

Learning Activity	Weighting	Estimated time a student should dedicate to prepare for and participate in
Lectures	26.7 %	40.0 hours
Discussions	13.3 %	20.0 hours
Exercises in class, Asynchronous sessions, Field Work	6.7 %	10.0 hours
Group work	13.3 %	20.0 hours
Individual studying	40.0 %	60.0 hours
TOTAL	100.0 %	150.0 hours

AI POLICY

Generative artificial intelligence (GenAI) tools may be used in this course for some specific tasks/assignments with appropriate acknowledgment. **AI is not allowed for text generation in assignments.** If a student is found to have used AI-generated content inappropriately, it will be considered academic misconduct, and the student might fail the respective assignment or the course.

If you are in doubt as to whether you are using GenAI tools appropriately in this course, discuss your situation with the professor.

Below, is a suggested format to acknowledge the use of generative AI tools. Please note that acknowledging AI will not impact your grade.

I acknowledge the use of [AI systems link] to [specify how you used generative AI]. The prompts used include [list of prompts]. The output of these prompts was used to [explain how you used the outputs in your work]

If AI was permitted to be used in your assignment, but you have chosen not to include any AI-generated content, the following disclosure is recommended:

No content generated by AI technologies has been used in this assignment.

PREREQUISITES

You will benefit from a strong understanding of vectors, matrices, planes, lines, and points in 3D. These foundational topics are essential to studying linear algebra, which involves manipulating and analyzing systems of linear equations and matrices. Throughout the course, you will build upon these concepts to explore more advanced topics and apply them to higher-dimensional vector spaces.

PROGRAM

Disclaimer: *The following description of the material covered is tentative. While an attempt will be made to cover all listed topics and include other advanced topics that will help students throughout their careers in applied mathematics, the pace of the classes will depend on group performance, which may introduce some variations in the syllabus.*

Pre and Post-work for each session:

- Necessary readings are described in each session's description. You will also find additional resources posted on Blackboard.

- Selected exercises, homework, and assignements will be indicated at the end of each session.

The course is organized into the following modules:

- Introduction
- Vector spaces and subspaces
- Matrices and linear transformations
- Determinants
- Diagonalization
- Jordan form and Singular Value Descomposition

SESSION 1 (LIVE IN-PERSON)

Course introduction, introduction to LaTeX, matrix-vector multiplication, and Gaussian elimination.

Please make sure you are comfortable with the following concepts:

- Vectors and their operations: https://www.3blue1brown.com/lessons/vectors

SESSIONS 2 - 3 (LIVE IN-PERSON)

Geometric interpretation of systems of linear equations. How to find the solutions when there is one unique solution.

Classification of linear equations and how to find their solutions. Rouché-Frobenius theorem.

Recommended readings:

- Please check documents and videos posted on Blackboard.
- Sections 1.1-1.3 (up to elementary matrices) and 1.8 from Applied Linear Algebra by P. J. Olver and C. Shakiban. Note: these sections include matrix arithmetics (can be a nice review or preview for matrix multiplication that we will see in upcoming sessions) and the factorization PA=LU (we will not cover this).

SESSION 4 (LIVE IN-PERSON)

Vector spaces, vector subspaces, span.

Previous work:

- Please read and watch: https://www.3blue1brown.com/lessons/abstract-vector-spaces

Recommended readings:

- Sections 1.1-1.3 from Linear Algebra by S. H. Friedberg, A. J. Insel, and L.E. Spence.
- Sections 2.1 and 2.2. from Applied Linear Algebra by P. J. Olver and C. Shakiban

SESSION 5 (LIVE IN-PERSON)

Vector spaces, vector subspaces, span (cont.) and problems.

Recommended readings:

- Sections 1.1-1.3 from Linear Algebra by S. H. Friedberg, A. J. Insel, and L.E. Spence.
- Sections 2.3 from Applied Linear Algebra by P. J. Olver and C. Shakiban

SESSION 6 (LIVE IN-PERSON)

Linear independence; sum, intersection, and union of subspaces.

Previous work:

- Read and watch the following to review and prepare for this session: https://www.3blue1brown.com/lessons/span

Recommended readings:

- Section 2.3 from Applied Linear Algebra by P. J. Olver and C. Shakiban.
- Section 1.5 from Linear Algebra by S. H. Friedberg, A. J. Insel, and L.E. Spence.

SESSIONS 7 - 8 (LIVE IN-PERSON)

Replacement theorem, basis, dimension, and problems.

Please check materials available on Blackboard. Recommended readings:

- Section 1.6 from Linear Algebra by S. H. Friedberg, A. J. Insel, and L.E. Spence.

SESSION 9 (LIVE IN-PERSON)

Linear transformations.

Previous work:

- Read and watch: https://www.3blue1brown.com/lessons/linear-transformations
- Read and watch: https://www.3blue1brown.com/lessons/3d-transformations
- Read and watch: https://www.3blue1brown.com/lessons/nonsquare-matrices

Recommended reading:

- Section 2.1 from Linear Algebra by S. H. Friedberg, A. J. Insel, and L.E. Spence

SESSION 10 (LIVE IN-PERSON)

Kernel and Image.

Recommended readings:

- Section 2.1 and 2.2 from Linear Algebra by S. H. Friedberg, A. J. Insel, and L.E. Spence

SESSION 11 (LIVE IN-PERSON)

Kernel and Image (cont.). Problems.

Please check additional materials posted on Blackboard.

SESSION 12 (LIVE IN-PERSON)

Quiz 1

SESSION 13 (LIVE IN-PERSON)

Matrices: operations and properties.

Previous work:

- Read and watch the following if you are unfamiliar with matrix multiplication: https://www.3blue1brown.com/lessons/matrix-multiplication

Recommended readings:

- Chapter 5 from Practical Linear Algebra for Data Science by Mike X Cohen.

SESSION 14 (LIVE IN-PERSON)

Isomorphisms and invertibility.

Previous work:

- Read and watch (some things are new, others will help you review concepts): https://www.3blue1brown.com/lessons/inverse-matrices

Recommended readings:

- Go through example 1.23 on Gauss-Jordan from Applied Linear Algebra by P. J. Olver and C. Shakiban
- Section 2.4 from Linear Algebra by S.H. Friedberg, A. J. Insel, and L.E. Spence.

SESSION 15 (LIVE IN-PERSON)

Change of basis.

Previous work:

- Read and watch: https://www.3blue1brown.com/lessons/change-of-basis

Recommended readings:

- "Change of basis" section in chapter 7.2 Applied Linear Algebra by P. J. Olver and C. Shakiban.
- Section 2.5 from Linear Algebra by S.H. Friedberg, A. J. Insel, and L.E. Spence.

SESSION 16 (LIVE IN-PERSON)

Linear Algebra challenge.

SESSION 17 (LIVE IN-PERSON)

Determinants (geometrical interpretation).

Previous work:

- Please read and watch:https://www.3blue1brown.com/lessons/determinant

Recommended readings:

- Section 1.9 from Applied Linear Algebra by J.P. Olver and C. Shakiban.
- Chapter 4 (or at least section 4.4) from Linear Algebra by S. H. Friedberg, A. J. Insel, and L.E. Spence.

SESSION 18 (LIVE IN-PERSON)

Determinants (algebraic description).

Previous work:

- Please read and watch: https://www.3blue1brown.com/lessons/cramers-rule

Recommended readings:

- Section 1.9 from Applied Linear Algebra by J.P. Olver and C. Shakiban.
- Chapter 4 (or at least section 4.4) from Linear Algebra by S. H. Friedberg, A. J. Insel, and L.E. Spence.

SESSION 19 (LIVE IN-PERSON)

Midterm review

SESSION 20 (LIVE IN-PERSON)

Midterm

SESSION 21 (LIVE IN-PERSON)

Introduction to diagonalization.

Previous work:

- Read and watch: https://www.3blue1brown.com/lessons/eigenvalues

Recommended readings:

- Section 8.2 from Applied Linear Algebra by J.P. Olver and C. Shakiban.
- Section 5.1 from Linear Algebra by S.H. Friedberg, A. J. Insel, and L.E. Spence.

SESSION 22 (LIVE IN-PERSON)

Characteristic polynomial, mulltiplicty and test for diagonalizability.

Previous work:

- How to factorize a polynomial using Ruffini's technique, please watch: https://www.youtube.com/watch?v=ZofBCD1Ts1g

Recommended readings:

- Section 8.3 from Applied Linear Algebra by J.P. Olver and C. Shakiban.
- Section 5.2 from Linear Algebra by S.H. Friedberg, A. J. Insel, and L.E. Spence.

SESSION 23 (LIVE IN-PERSON)

Cayley-Hamilton Theorem.

After class, please review the material posted on Blackboard.

SESSION 24 (LIVE IN-PERSON)

Cayley-Hamilton Theorem and problems.

Recommended readings:

- Section "Cayley-Hamilton Theorem" within 5.4 from Linear Algebra by S.H. Friedberg, A. J. Insel, and L.E. Spence.

SESSION 25 (LIVE IN-PERSON)

Linear Algebra applications: PageRank (part 1).

SESSION 26 (LIVE IN-PERSON)

Linear Algebra applications: PageRank (part 2).

SESSION 27 (LIVE IN-PERSON)

Jordan canonical form.

Recommended readings:

- Sections 7.1 and 7.2 from Linear Algebra by S. H. Friedberg, A. J. Insel, and L.E. Spence.

SESSION 28 (LIVE IN-PERSON)

Jordan canonical form, the minimal polynomial, and problems.

Recommended readings:

- Sections 7.1 7.2, and 7.3 from Linear Algebra by S. H. Friedberg, A. J. Insel, and L.E. Spence.

SESSION 29 (LIVE IN-PERSON)

Quiz 2

SESSION 30 (LIVE IN-PERSON)

Project presentations.

SESSION 31 (LIVE IN-PERSON)

Singular Value Decomposition (SVD).

You can review the concepts (and beyond) here: https://databookuw.com/databook.pdf

The link is the.pdf of the book "Data Driven Science & Engineering" by S.L. Brunton and J.N. Kutz.

SESSION 32 (LIVE IN-PERSON)

SVD problems and eigenfaces.

After the sessions you can watch the following video about SVD and eigenfaces (with the Yale Data base): https://www.youtube.com/watch?v=ofWji_wQBEE&list=PLMrJAkhIeNNSVjnsviglFoY2nXildDCcv

SESSION 33 (LIVE IN-PERSON)

Review

SESSION 34 (LIVE IN-PERSON)

Review

SESSION 35 (LIVE IN-PERSON)

Final exam

EVALUATION CRITERIA

criteria	percentage	Learning Objectives	Comments

Final exam	45 %		It is worth 45% of the overall grade. You need to score at least 3.5 on the final exam to pass the overall course, even if you have already passed the course through the other course assessments.
Intermediate tests	30 %		The midterm exam & quizzes will account for 30% of the overall grade. Please ensure you are prepared for these assessments.
Group Project	15 %		You will carry out a group project with other classmates which will be 15% of the overall grade.
Class Participation	10 %		Webwork problems. Each module in the course has its own Webwork's problem set, which includes several exercises. Students are expected to work on these exercises and submit them (deadlines will be announced throughout the course). Active participation during lectures is also expected, with students encouraged to ask questions and make remarks. The class grade will be based on punctuality, participation, and class conduct. Please note that there may be a penalty for disruptions or excessive talking during class.

RE-SIT / RE-TAKE POLICY

Each student has four chances to pass any given course distributed over two consecutive academic years: ordinary call exams and extraordinary call exams (re-sits) in June/July. Students who do not comply with the 80% attendance rule during the semester will fail both calls for this Academic Year (ordinary and extraordinary) and have to re-take the course (i.e., re-enroll) in the next Academic Year.

Evaluation criteria:

- Students failing the course in the ordinary call (during the semester) will have to re-sit the exam in June / July (except those not complying with the attendance rule, who will not have that opportunity and must directly re-enroll in the course on the next Academic Year).
- The extraordinary call exams in June / July (re-sits) require your physical presence at the campus you are enrolled in (Segovia or Madrid). There is no possibility to change the date, location or format of any exam, under any circumstances. Dates and location of the June / July re-sit exams will be posted in advance. Please consider this when planning your summer.
- The June / July re-sit exam will consist of a comprehensive exam. Your final grade for the course will depend on your performance in this exam only; continuous evaluation over the semester will not be taken into consideration. Students will have to achieve the minimum passing grade of 5 and can obtain a maximum grade of 8.0 (out of 10.0) – i.e., “notable” in the re-sit exam.
- Retakers: Students who failed the subject on a previous Academic Year and are now re-enrolled as re-takers in a course will be needed to check the syllabus of the assigned professor, as well as contact the professor individually, regarding the specific evaluation criteria for them as retakers in the course during that semester (ordinary call of that Academic Year). The maximum grade that may be obtained in the retake exam (3rd call) is 10.0.
  After the professor grades ordinary and extraordinary call exams, you will have the possibility to attend a review session for that exam and course grade. Please be available to attend the session in order to clarify any concerns you might have regarding your exam. Your professor will inform you about the time and place of the review session. Any grade appeals require that the student attended the review session prior to appealing.

Students failing more than 18 ECTS credits (BAM) or 21 ECTS credits (BIEBAM) after the June / July re-sits will be asked to leave the Program. Please, make sure to prepare yourself well for the exams in order to pass your failed subjects.

In case you decide to skip the opportunity to re-sit for an exam during the June/July extraordinary call, you will need to enroll in that course again for the next Academic Year as a re-taker and pay the corresponding extra cost. As you know, students have a total of four allowed calls to pass a given subject or course, in order to remain in the program

BIBLIOGRAPHY

Compulsory

- Stephen Friedberg, Arnold Insel, Lawrence Spence. (2018). *Linear Algebra.* 5th. Pearson. ISBN 0134860241 (Digital)

Recommended

- Peter J. Olver, Chehrzad Shakiban. *Applied Linear Algebra.* ISBN 9783319910413 (Digital)
- Mike Cohen. *Practical Linear Algebra for Data Science: From Core Concepts to*

Applications Using Python. ISBN 1098120612 (Digital)
- Gilbert Strang. *Introduction to Linear Algebra.* ISBN 9781733146678 (Digital)
- Mike X Cohen. *Linear Algebra: Theory, Intuition, Code.* Sincxpress Education SRL. ISBN 9083136604 (Digital)
- W. Keith Nicholson. *Linear Algebra with Applications.* ISBN 1717015220 (Digital)
- Steven L. Brunton, J. Nathan Kutz. (2022). *Data Driven Science & Engineering.* 2. Cambridge University Press. ISBN 1009098489 (Digital)
https://databookuw.com/databook.pdf

BEHAVIOR RULES

Please, check the University's Code of Conduct here. The Program Director may provide further indications.

ATTENDANCE POLICY

Please, check the University's Attendance Policy here. The Program Director may provide further indications.

ETHICAL POLICY

Please, check the University's Ethics Code here. The Program Director may provide further indications.