MATRICES & LINEAR TRANSFORMATIONS

Grado en Computación e Inteligencia Artificial / Bachelor in Computer Science and Artificial Intelligence BCSAI SEP-2025
MLT-CSAI.2.M.A

Area Mathematics

Number of sessions: 30

Academic year: 25-26

Degree course: SECOND

Number of credits: 6.0

Semester: 2º

Category: BASIC

Language: English

Professor: **IGNACIO ZUBIZARRETA BALLESTEROS**

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Bachelor's Degree in Mathematics and Bachelor of Business Administration, with 20 years of experience in AI-driven projects.

He spent 17 years at a startup focused on customer experience, where he applied speech recognition and mobile solutions. During this time, he served as Managing Director for four years, leading the creation and implementation of innovative products powered by AI technologies.

Subsequently, he contributed to the digital transformation of two Contact Centers, optimizing business processes through automation and intelligent systems.

Currently, he works as a Project Director, developing solutions based on Artificial Intelligence and Optimization to address complex business challenges using mathematical models.

He also teaches as an adjunct professor of Matrices and Linear Transformations in the BCSAI program and Numerical Linear Algebra in the BAM program at IE University.

Office Hours

Office hours will be on request. Please contact at:

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SUBJECT DESCRIPTION

Matrices and linear transformations are part of linear algebra, a fundamental area of mathematics. Linear algebra provides us with powerful tools to apply solutions in fields such as computer science, business, physics, economics, and engineering.

In linear algebra, concepts are as important as computations. In your career, computers will handle the calculations, but you will have to choose the algorithm, interpret the results, and explain and present the final conclusions to others. Without a solid knowledge of linear algebra, you could omit important information or give unrealistic results based on the incorrect application of an algorithm.

The aim of this course is to revisit the foundations of vectors and matrices that you might have encountered in school but may not have applied. We are not aiming to conduct a fundamental course where everything is done very rigorously. Instead, we will have extensive practice with these concepts and their applications to provide mathematical solutions in real life.

More than anything else, we want to develop your mathematical intuition. By the end of this course, students will be capable of abstracting concepts of linear algebra to apply mathematical solutions to real problems.

LEARNING OBJECTIVES

At the end of this course, you should be able to:

- Demonstrate knowledge and understanding of concepts related to vector and representation in plane and in space.
- Show proficiency in solving Linear Systems of Equations.
- Know basic properties of Vector Spaces and operators between Vector Spaces.
- Perform Matrix computations and use Matrix operations to efficiently solve linear systems and determinate the inverse of a Matrix.
- Demonstrate proficiency in using Gram-Schmidt process to construct Orthonormal basis
- Show proficiency in applying the properties of Determinants in solving related problems.
- Demonstrate proficiency in finding Eigenvalues and Eigenvectors of a matrix, and in determining if a matrix is diagonalizable.
- Demonstrate understanding of the concept of Linear Transformation and show proficiency in solving problems in Linear Transformation.
- Analyzing applications of linear algebra.

  In summary, you will be capable of analyzing different problems based on linear algebra, finding the algorithm solution to apply, write your own code and make calculations to find the solution.

TEACHING METHODOLOGY

IE University teaching method is defined by its collaborative, active, and applied nature. Students actively participate in the whole process to build their knowledge and sharpen their skills. Professor's main role is to lead and guide students to achieve the learning objectives of the course. This is done by engaging in a diverse range of teaching techniques and different types of learning activities such as the following:

Learning Activity	Weighting	Estimated time a student should dedicate to prepare for and participate in
Lectures	20.0 %	30.0 hours
Discussions	13.3 %	20.0 hours
Exercises in class, Asynchronous sessions, Field Work	20.0 %	30.0 hours
Group work	20.0 %	30.0 hours
Individual studying	26.7 %	40.0 hours
TOTAL	100.0 %	150.0 hours

AI POLICY

In today's world, generative artificial intelligence (GenAI) is changing how we work, study and, in general, how we get things done. However, in the context of this course, the use of GenAI is not permitted, unless it is otherwise stated by the instructor. The use of GenAI tools would jeopardize the students' ability to acquire fundamental knowledge or skills of this course.

If a student is found to have used AI-generated content for any form of assessment, it will be considered academic misconduct, and the student might fail the respective assignment or the course.

PROGRAM

MODULE 1: VECTORS AND MATRICES

SESSION 1 (LIVE IN-PERSON)

Introduction to Vectors

Vectors and Linear Combination

Length, Dot Product and Unit vectors

Minkowski distances

Cosine similarity

Vectors applications

SESSION 2 (LIVE IN-PERSON)

Matrix definition

Types and properties of Matrices

Transpose and inverse of a matrix

Matrix multiplication

Strassen Algorithm

Matrices exercises

SESSION 3 (LIVE IN-PERSON)

Practice: Review key ideas, Vectors and Matrices applications.

MODULE 2: SOLVING LINEAR EQUATIONS

SESSION 4 (LIVE IN-PERSON)

Linear equations (n equations, n unknowns)
Row picture and Column picture of equations in 2D and 3D
Matrix picture
The Idea of Gauss Elimination.

SESSION 5 (LIVE IN-PERSON)

Gauss Elimination using Matrices
Elimination Matrix
Permutation Matrix
Row Echelon Form (REF)
Gauss Jordan Elimination
Row Reduced Echelon Form (RREF)

SESSION 6 (LIVE IN-PERSON)

Factorization A=LU
Factorization A=LDU
Factorizations PA=LU
Computational cost A=LU

SESSION 7 (LIVE IN-PERSON)

Practice
Application Balance Chemistry
Application Leontieff
Application Electric circuit

SESSION 8 (LIVE IN-PERSON)

Gauss Jordan inverse of A
Round off error
Partial Pivoting
Network Flow Application

SESSION 9 (LIVE IN-PERSON)

Jacobi numerical method
Gauss Siedel numerical method

Finite Linear Games
GPS calculation

MODULE 3: VECTOR SPACES

SESSION 10 (LIVE IN-PERSON)

Vector spaces and Subspaces
Span. base and rank
Echelon Form and Pivots
Reduced Row Echelon Form
Column space of A: Solving Ax=b
Nullspace of A: Solving Ax=0

SESSION 11 (LIVE IN-PERSON)

Subspace dimension
Complete solution of Ax=b
Row Space of A
Nullspace of A transpose
The 4 fundamental subspaces picture

SESSION 12 (LIVE IN-PERSON)

Independence vectors
Basis of subspaces
Complete solution of Ax=b

SESSION 13 (LIVE IN-PERSON)

Subspaces
4 Fundamental Subspaces
Complete solution of Ax=b
Short questions

SESSION 14 (LIVE IN-PERSON)

MIDTERM REVIEW

SESSION 15 (LIVE IN-PERSON)

MIDTERM EXAM

MODULE 4: ORTHOGONALITY

SESSION 16 (LIVE IN-PERSON)

Orthogonal vectors
Orthogonal subspaces
Fundamental 4 subspaces orthogonality
Projections
Least Squares

SESSION 17 (LIVE IN-PERSON)

Orthonormal Bases
Orthogonal matrix Q
Gram-Schmidt
QR Decomposition

SESSION 18 (LIVE IN-PERSON)

Practice:
Least Squares for different equations
Gram Schmidt
QR Decomposition

MODULE 5: DETERMINANTS

SESSION 19 (LIVE IN-PERSON)

Determinant properties
Formula of determinant
Cofactors
Cross Product Application
Condensed method for determinants

SESSION 20 (LIVE IN-PERSON)

Formula for inverse of A
Cramer Rule
Areas
Volumes
Application: Hill Cipher

MODULE 6: EIGENVALUES AND EIGENVECTORS

SESSION 21 (LIVE IN-PERSON)

Introduction to Eigenvalues and Eigenvectors
Diagonalizing a matrix
Matrix powers via diagonalization

SESSION 22 (LIVE IN-PERSON)

Fibonacci sequence
Markov Matrices
Differential Equations

SESSION 23 (LIVE IN-PERSON)

Math Challenge: Team Competition

SESSION 24 (LIVE IN-PERSON)

Symmetric Matrices
Singular Value Decomposition SVD

SESSION 25 (LIVE IN-PERSON)

Practice:
Singular Value Decomposition SVD
Principal Component Analysis

MODULE 7: LINEAR TRANSFORMATIONS

SESSION 26 (LIVE IN-PERSON)

Linear Transformations
Matrix of Linear Transformations
Inverse of Transformation
Composition of Linear Transformations

SESSION 27 (LIVE IN-PERSON)

Practice: Review Key ideas, Linear Transformations

SESSION 28 (LIVE IN-PERSON)

Presentation of Linear Algebra Applications by students.

SESSION 29 (LIVE IN-PERSON)

Final review session: Vectors and Matrices, Solving Linear Equations, Vector Spaces, Orthogonality, Determinants, Eigenvalues and Linear Transformations

SESSION 30 (LIVE IN-PERSON)

FINAL EXAM

EVALUATION CRITERIA

A. CLASS PARTICIPATION

It will be worth 10% of the overall grade. The class participation evaluation aims to assess students' active engagement during lectures, fostering an interactive and collaborative learning environment. Students are expected to contribute meaningfully to discussions, ask questions, provide relevant remarks, participate in class exercises, and engage in work group challenges. Active participation is not only measured by the frequency of contributions but also by the quality and relevance of the student's participation in class activities.

B. GROUP WORK

The students will be organized in groups. Every group will work on a particular application in Linear Algebra selected from the modules of the program. This work will be graded after the deadline and your overall "Group Work" grade is worth 30% of the final grade.

C. INTERMEDIATE TESTS: EXERCISES + MIDTERM.

It is worth 30% of the final grade. We will have exercises assignments in this course. They will be based on exercises similars to the ones we will see in class.

D. FINAL-EXAM

It is worth 30% of the overall grade. You need to score at least 3.5 on the final exam to pass the overall course, even if you have already passed the course through the other course assessments. Information about the detailed characteristics of the final-exam will be given at the beginning of the semester.

criteria	percentage	Learning Objectives	Comments
Final Exam	30 %		
Group Work	30 %		
Individual work	30 %		Assignments + Midterm Exam
Class Participation	10 %		

RE-SIT / RE-TAKE POLICY

Each student has four chances to pass any given course distributed over two consecutive academic years: ordinary call exams and extraordinary call exams (re-sits) in June/July.

Students who do not comply with the 80% attendance rule during the semester will fail both calls for this Academic Year (ordinary and extraordinary) and have to re-take the course (i.e., re-enroll) in the next Academic Year.

Evaluation criteria:

- Students failing the course in the ordinary call (during the semester) will have to re-sit the exam in June / July (except those not complying with the attendance rule, who will not have that opportunity and must directly re-enroll in the course on the next Academic Year).
- The extraordinary call exams in June / July (re-sits) require your physical presence at the campus you are enrolled in (Segovia or Madrid). There is no possibility to change the date, location or format of any exam, under any circumstances. Dates and location of the June / July re-sit exams will be posted in advance. Please take this into consideration when planning your summer.

- The June / July re-sit exam will consist of a comprehensive exam. Your final grade for the course will depend on the performance in this exam only; continuous evaluation over the semester will not be taken into consideration. Students will have to achieve the minimum passing grade of 5 and can obtain a maximum grade of 8.0 (out of 10.0) - i.e., "notable" in the in the re-sit exam.
- Retakers: Students who failed the subject on a previous Academic Year and are now re-enrolled as re-takers in a course will be needed to check the syllabus of the assigned professor, as well as contact the professor individually, regarding the specific evaluation criteria for them as retakers in the course during that semester (ordinary call of that Academic Year). The maximum grade that may be obtained in the retake exam (3rd call) is 10.0.

After ordinary and extraordinary call exams are graded by the professor, you will have a possibility to attend a review session for that exam and course grade. Please be available to attend the session in order to clarify any concerns you might have regarding your exam. Your professor will inform you about the time and place of the review session. Any grade appeals require that the student attended the review session prior to appealing.

- Students failing more than 18 ECTS credits in the academic year after the June-July re-sits will be asked to leave the Program. Please, make sure to prepare yourself well for the exams in order to pass your failed subjects.
- In case you decide to skip the opportunity to re-sit for an exam during the June / July extraordinary call, you will need to enroll in that course again for the next Academic Year as a re-taker and pay the corresponding extra cost. As you know, students have a total of four allowed calls to pass a given subject or course, in order to remain in the program.

BIBLIOGRAPHY

Recommended

- Gilbert Strang. (2020). *Linear Algebra for Everyone.* Wellesley-Cambridge Press. ISBN 9781733146630 (Printed)
- Howard Anton. (2019). *Elementary Linear Algebra.* Wiley. ISBN 9781119406723 (Printed)

BEHAVIOR RULES

Please, check the University's Code of Conduct here. The Program Director may provide further indications.

ATTENDANCE POLICY

Please, check the University's Attendance Policy here. The Program Director may provide further indications.

ETHICAL POLICY

Please, check the University's Ethics Code here. The Program Director may provide further indications.